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distortions for the compensation regime Three−**dimensional flow near surface**

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I. I. Lipatov and I. V. Vinogradov

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$\frac{10.1098/\text{rsta}.2000.0701}{\text{Three-dimensional flow near surface distortions}}$ mensional flow near surface dister-
for the compensation regime for the compensation regime
BY I. I. LIPATOV¹ AND I. V. VINOGRADOV²

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 16 Gagarin Street, Zhukovsky, Moscow region, 140160, Russia

Flow past an isolated, small, three-dimensional roughness located on a plate is inves-
Flow past an isolated, small, three-dimensional roughness located on a plate is inves-
tigated theoretically and numerically. Depending Flow past an isolated, small, three-dimensional roughness located on a plate is inves-
tigated theoretically and numerically. Depending on roughness scales, there are differ-
ent local disturbed flow regimes. The so-called Flow past an isolated, small, three-dimensional roughness located on a plate is inves-
tigated theoretically and numerically. Depending on roughness scales, there are differ-
ent local disturbed flow regimes. The so-called tigated theoretically and numerically. Depending on roughness scales, there are different local disturbed flow regimes. The so-called 'compensation regime' is characterized by conservation of boundary-layer thickness for w ent local disturbed flow regimes. The so-called 'compensation regime' is characterized
by conservation of boundary-layer thickness for which the flow near the roughness
does not interact with the external inviscid flow. A by conservation of boundary-layer thickness for which the flow near the redoes not interact with the external inviscid flow. A spectral numerical m used to calculate the corresponding three-dimensional boundary problem.

used to calculate the corresponding three-dimensional boundary problem.
Keywords: fluid dynamics; high Reynolds number; surface distortions
three-dimensional boundary-layer flows Keywords: fluid dynamics; high Reynolds number; surface distortions;

1. Introduction

1. Introduction
Phenomena arising in the flow near surface distortions play a significant role in
geophysical flows in external and internal aerodynamics etc. Modern trends in The cause of the momentum corrections.
The peophysical flows, in external and internal aerodynamics, etc. Modern trends in
houndary-layer flow control give the basis of a more detailed analysis. The laminargeophysical flows, in external and internal aerodynamics, etc. Modern trends in boundary-layer flow control give the basis of a more detailed analysis. The laminargeophysical flows, in external and internal aerodynamics, etc. Modern trends in
boundary-layer flow control give the basis of a more detailed analysis. The laminar-
turbulent transition, skin friction and heat transfer dis boundary-layer flow control give the basis of a more detailed analysis. The laminar-
turbulent transition, skin friction and heat transfer distributions depend strongly on
surface irregularities. By means of controlled uns turbulent transition, skin friction and heat transfer distributions depend strongly of surface irregularities. By means of controlled unsteady motion of surface irregularities the transition may be delayed or boundary-lay surface irregularities. By means of controlled unsteady motion of surface irregularities
the transition may be delayed or boundary-layer separation may be suppressed.
Different local flow regimes are described by Smith (19

the transition may be delayed or boundary-layer separation may be suppressed.
Different local flow regimes are described by Smith (1973, 1976), Bogolepov & Nei-
land (1971), Bogolepov (1986), Duck & Burggraf (1986), Bogole Different local flow regimes are described by Smith (1973, 1976), Bogolepov & Neiland (1971), Bogolepov (1986), Duck & Burggraf (1986), Bogolepov & Lipatov (1985) and Sykes (1980). We do not aim to discuss all aspects of l land (1971), Bogolepov (1986), Duck & Burggraf (1986), Bogolepov & Lipatov (1985)
and Sykes (1980). We do not aim to discuss all aspects of locally disturbed flows. A
more comprehensive list of publications is presented, and Sykes (1980). We do not aim to discuss all aspects of locally disturbed flows. A more comprehensive list of publications is presented, for example, by Smith & Walton (1998) and by Roget *et al.* (1998). Investigations more comprehensive list of publications is presented, for example, by Smith $\&$ Walton (1998) and by Roget *et al.* (1998). Investigations of two-dimensional flows near surface distortions provided by Bogolepov & Neiland (1971), analysis of pipeflows
done by Smith (1976) and stratified flows studies made by Sykes (1980) allow us to
find the so-called compensation regime. This regime is c done by Smith (1976) and stratified flows studies made by Sykes (1980) allow us to find the so-called compensation regime. This regime is characterized by conservation
of boundary-layer thickness and corresponds to a variety of disturbed flows.
At the same time the numerical solutions for nonlinear probl

At the same time the numerical solutions for nonlinear problems were obtained. At the same time the numerical solutions for nonlinear problems were obtained
by Sykes (1980) only for flows periodical in the transverse direction. Not all the
problems were solved from a theoretical point of view. This p by Sykes (1980) only for flows periodical in the transverse direction. Not all the problems were solved from a theoretical point of view. This paper is devoted to the analysis of mathematical and physical aspects of locall problems were solved from a theoretical point of view. This paper is devoted to the analysis of mathematical and physical aspects of locally disturbed flows. The aim is to describe the main processes responsible for distur formulate a well-posed mathematical and physical aspects of locally disturbed flows. The aim
is to describe the main processes responsible for disturbance propagation and to
formulate a well-posed mathematical problem desc to describe the main processes responsible for disturbance propagation and to rmulate a well-posed mathematical problem describing the compensation regime.
This paper is also devoted to the numerical investigation of the f

formulate a well-posed mathematical problem describing the compensation regime.
This paper is also devoted to the numerical investigation of the flow near three-
dimensional surface distortion for the compensation regime.

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2. Problem formulation

2. Problem formulation
The flow near the flat plate is considered. It is supposed that the small surface
distortion is located on the bottom of a laminar two-dimensional boundary layer at The flow near the flat plate is considered. It is supposed that the small surface
distortion is located on the bottom of a laminar two-dimensional boundary layer at
a distance l from the flat plate leading edge. It is supp distortion is located on the bottom of a laminar two-dimensional boundary layer at a distance l from the flat plate leading edge. It is supposed that the external flow is distortion is located on the bottom of a laminar two-dimensional boundary layer at a distance l from the flat plate leading edge. It is supposed that the external flow is uniform subsonic or supersonic viscous flow $(M_{\infty$ a distance l from the flat plate leading edge. It is supposed that the external flow is
uniform subsonic or supersonic viscous flow $(M_{\infty}^2 - 1) \sim 1$ for large but subcritical
Reynolds numbers $Re_{\infty} = \rho_{\infty} u_{\infty} l/\mu_{\in$ uniform subsonic or supersonic viscous flow $(M_{\infty}^2 - 1) \sim 1$ for large but subcritical
Reynolds numbers $Re_{\infty} = \rho_{\infty} u_{\infty} l/\mu_{\infty} = \varepsilon^{-2}$; ρ , u and μ are the density, the velocity
and the dynamical viscosit Reynolds numbers $Re_{\infty} = \rho_{\infty} u_{\infty} l/\mu_{\infty} = \varepsilon^{-2}$; ρ , u and μ are the density, the velocity
and the dynamical viscosity coefficient, respectively; the index ∞ corresponds to
the values in the external fl and the dynamical viscosity coefficient, respectively; the index ∞ corresponds to the values in the external flow. Cartesian coordinates are introduced, where X, Y and Z are the streamwise, normal and spanwise axes, r the values in the external flow. Cartesian coordinates are introduced, where X, Y
and Z are the streamwise, normal and spanwise axes, respectively. We will use the
following definitions below: lt/u_{∞} , lx , ly , lz , u and Z are the streamwise, normal and spanwise axes, respectively. We will use the following definitions below: lt/u_{∞} , lx , ly , lz , $u_{\infty}u$, $u_{\infty}v$, $u_{\infty}w$, $\rho_{\infty}u_{\infty}^2$, $\rho_{\infty}\rho$, $\mu_{\infty}\mu$ for time, following definitions below: lt/u_{∞} , lx , ly , lz , $u_{\infty}u$, $u_{\infty}v$, $u_{\infty}w$, $\rho_{\infty}u_{\infty}^2$, $\rho_{\infty}\rho$, $\mu_{\infty}\mu$ for time, coordinates, velocity components, pressure, density and dynamical viscosity coeff time, coordinates, velocity components, pressure, density and dynamical viscosity coefficient, respectively. It is supposed that the distortion has the scale-thickness a , length b and width c .
Let us consider the fl

length b and width c.
Let us consider the flow near the local distortion having comparable length and
width $b \sim c$. It is evident that the limiting problems for asymptotically different
values characterizing the width and Let us consider the flow near the local distortion having comparable length and width $b \sim c$. It is evident that the limiting problems for asymptotically different values characterizing the width and the length may be obt width $b \sim c$. It is evident that the limiting problems for asymptotically different values characterizing the width and the length may be obtained as a result of the corresponding limiting procedure from the general mathe gated. It is also supposed that the characteristic length and the width of the distortion
It is also supposed that the characteristic length and the width of the distortion

gated.
It is also supposed that the characteristic length and the width of the distortion
are much larger than the undisturbed boundary-layer thickness in the vicinity of
the distortion. In accordance with the matched asym It is also supposed that the characteristic length and the width of the distortion are much larger than the undisturbed boundary-layer thickness in the vicinity of the distortion. In accordance with the matched asymptotic are much larger than the undisturbed boundary-layer thickness in the vicinity of
the distortion. In accordance with the matched asymptotic expansions method, we
need to introduce the region 1 having asymptotically equal sc the distortion. In accordance with the matched asymptotic expansions method, we here the region 1 having asymptotically equal scales in all dimensions. It may be shown for small thickness values that the disturbed flow is characterized
by equal values of disturbances for velocity components, pressure and density, which
are determined by the vertical velocity value on the by equal values of disturbances for velocity components, pressure and density, which are determined by the vertical velocity value on the external edge of the boundary layer. This required value may be estimated as the rat are determined by the vertical velocity value on the external edge of the boundary

$$
\Delta p \sim v \sim a/b. \tag{2.1}
$$

This estimate allows us to determine the boundary-layer thickness change induced $\Delta p \sim v \sim a/v.$ (2.1)
This estimate allows us to determine the boundary-layer thickness change induced
by the pressure disturbance. The longitudinal velocity near the distortion in the
undisturbed boundary layer at a height This estimate allows us to determine the boundary-layer thickness change induced
by the pressure disturbance. The longitudinal velocity near the distortion in the
undisturbed boundary layer at a height comparable with that by the pressure disturbance. The longitudisturbed boundary layer at a height determined by the following expression:

$$
u \sim a/\varepsilon. \tag{2.2}
$$

 $u \sim a/\varepsilon$. (2.2)
If the distortion induces nonlinear longitudinal velocity changes, then the following
relations are valid If the distortion indu
relations are valid,

$$
u \sim \Delta u \sim \Delta p^{1/2},\tag{2.3}
$$

$$
y \sim \Delta y \sim \varepsilon \Delta p^{1/2},\tag{2.4}
$$

where the last relation determines the nonlinear region thickness. The vertical veloc $y \sim \Delta y \sim \epsilon \Delta p$, (2.4)
where the last relation determines the nonlinear region thickness. The vertical veloc-
ity on the external boundary-layer edge is induced both by the distortion height and
the change in boundary-lay where the last relation determines the nonlinear region thickness. The vertical velocity on the external boundary-layer edge is induced both by the distortion height and the change in boundary-layer thickness. It may then ity on the external boundary-layer edge is induced both by the distortion height and
the change in boundary-layer thickness. It may then be shown that the total change
in boundary-layer thickness is determined to leading o the change in boundary-layer thickness. It may then be shown that the total change
in boundary-layer thickness is determined to leading order by the region located
near the wall where streamlines have nonlinear changes in in boundary-layer thickness is determined to leading order
near the wall where streamlines have nonlinear changes in los
pressure disturbance estimate may be written in the form:

$$
\Delta p \sim (a/b) + \varepsilon \Delta p^{1/2} / b. \tag{2.5}
$$

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As was shown by Bogolepov & Neiland (1971), this relation is valid only if

$$
b/a \sim b^2/\varepsilon^2. \tag{2.6}
$$

The other limit, corresponding to

$$
b^2/\varepsilon^2 \ll b/a,\tag{2.7}
$$

leads to the discrepancy that the change in boundary-layer thickness induces a much leads to the discrepancy that the change in boundary-layer thickness induces a much
larger pressure disturbance than the original pressure disturbance leading to the
thickness change. The problem may be resolved if disturb leads to the discrepancy that the change in boundary-layer thickness induces a much
larger pressure disturbance than the original pressure disturbance leading to the
thickness change. The problem may be resolved if disturb larger pressure disturbance than the original pressure disturbance leading to the thickness change. The problem may be resolved if disturbances in the external flow are absent (to leading order). Therefore, instead of rela thickness change. The problem may be resolved if disturbances in the external flow are absent (to leading order). Therefore, instead of relation (2.5) , the following compensation relation (zero change in total boundary-

$$
a/b \sim \varepsilon \Delta p^{1/2}/b,\tag{2.8}
$$

$$
\Delta p \sim a^2/\varepsilon^2. \tag{2.9}
$$

It is necessary to also take into account equal orders of viscosity and inertia forces in It is necessary to also take into account equal orders of viscosity and inertia forces in nonlinearly disturbed region. This condition follows from the longitudinal momentum equation analysis It is necessary to als
nonlinearly disturbe
equation analysis,

$$
a \sim \varepsilon b^{1/3}.\tag{2.10}
$$

All estimates obtained earlier lead to the inequality,

$$
b \ll \varepsilon^{3/4},\tag{2.11}
$$

 $b \ll \varepsilon^{3/4}$, (2.11)
determining the length of distortion when compensation regime take place. The
equality in (2.11) corresponds to the free interaction regime determining the length of distortion when compensation reguality in (2.11) corresponds to the free interaction regime.
The conservation of boundary-layer thickness may be written termining the length of distortion when compensation regime take place
uality in (2.11) corresponds to the free interaction regime.
The conservation of boundary-layer thickness may be written in the form,

$$
u = Ay\varepsilon^{-1} + o(1),\tag{2.12}
$$

where A is the non-dimensional skin friction on the surface in the undisturbed boundwhere A is the non-dimensional skin friction on the surface in the undisturbed bound-
ary layer upstream from the distortion. This condition follows from the matching
procedure applied to solutions in the region 2 (the where A is the non-dimensional skin friction on the surface in the undisturbed bound-
ary layer upstream from the distortion. This condition follows from the matching
procedure applied to solutions in the region 2 (the ma ary layer upstream from the distortion. This condition follows from the procedure applied to solutions in the region 2 (the main part of the bot flow) and in the region 3 (nonlinearly disturbed region near the wall). flow) and in the region 3 (nonlinearly disturbed region near the wall).
3. Boundary value problem

3. Boundary value problem
In region 3, which has length-scales $x \sim b$, $y \sim \varepsilon b^{1/3}$, $z \sim c$, the following asymptotic
expansions are introduced: In region 3, which has length
expansions are introduced: expansions are introduced:

$$
\text{pansions are introduced:} \\
 x = bx_3, \quad y = \varepsilon b^{1/3} \mu_{\rm w}^{1/3} \rho_{\rm w}^{-1/3} A^{-1/3} y_3, \quad z = cz_3,\n \tag{3.1 a}
$$

$$
u = A^{1/3} \mu_{\rm w}^{1/3} \rho_{\rm w}^{-1/3} b^{1/3} u_3 + \cdots, \tag{3.1b}
$$

$$
v = A^{1/3} \mu_{\rm w}^{2/3} \rho_{\rm w}^{-2/3} \varepsilon b^{-1/3} v_3 + \cdots, \qquad (3.1c)
$$

$$
w = A^{2/3} \mu_{\rm w}^{1/3} \rho_{\rm w}^{-1/3} b^{-2/3} c w_3 + \cdots , \qquad (3.1 d)
$$

$$
w = A^{2/3} \mu_{w}^{1/3} \rho_{w}^{-1/3} b^{-2/3} c w_{3} + \cdots,
$$
\n(3.1 d)
\n
$$
p = 1/\gamma M_{\infty}^{2} + A^{4/3} \mu_{w}^{2/3} \rho_{w}^{1/3} b^{2/3} p_{3} + \cdots,
$$
\n
$$
\rho = \rho_{w} + \cdots, \quad \mu = \mu_{w} + \cdots.
$$
\n(3.1 e)

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The limiting relations, $a \sim \varepsilon b^{1/3}$, ε
following equations deduced from the $, \ \varepsilon^{3/2}$ v and I. V. Vinogradov
 $3/2 < b < \varepsilon^{3/4}$, $c \sim b$, then correspond to the
he Navier-Stokes equations for leading terms in following equations, deduced from the Navier-Stokes equations for leading terms in (3.1):

$$
\frac{\partial u_3}{\partial x_3} + \frac{\partial v_3}{\partial y_3} + \frac{\partial w_3}{\partial z_3} = 0, \qquad (3.2 a)
$$

$$
u_3 \frac{\partial u_3}{\partial x_3} + v_3 \frac{\partial u_3}{\partial y_3} + w_3 \frac{\partial u_3}{\partial z_3} + \frac{\partial p_3}{\partial x_3} = \frac{\partial^2 u_3}{\partial y_3^2},
$$
(3.2*b*)

$$
\frac{\partial x_3}{\partial y_3} = \frac{\partial y_3}{\partial y_3},
$$
\n(3.20)\n
\n
$$
\frac{\partial p_3}{\partial y_3} = 0,
$$
\n(3.2c)

$$
u_3 \frac{\partial w_3}{\partial x_3} + v_3 \frac{\partial w_3}{\partial y_3} + w_3 \frac{\partial w_3}{\partial z_3} + D \frac{\partial p_3}{\partial z_3} = \frac{\partial^2 w_3}{\partial y_3^2},
$$
(3.2 d)

 $u_3 \frac{\partial u_3}{\partial x_3} + v_3 \frac{\partial u_3}{\partial y_3} + w_3 \frac{\partial u_3}{\partial y_3}$
where the similarity parameter $D = b^2/c^2$
width Boundary conditions include the us $\frac{c^2}{c^2}$ is $i_3 + D \overline{\partial z_3} = \overline{\partial y_3^2}$, (3.2 a)
is determined by the ratio of length to
the conditions for the three-dimensional where the similarity parameter $D = b^2/c^2$ is determined by the ratio of length to width. Boundary conditions include the usual conditions for the three-dimensional boundary layer along with total zero thickness change con where the similarity parameter $D = b^2/c^2$ is determined by the width. Boundary conditions include the usual conditions for the thoundary layer along with total zero thickness change condition, with total zero thickness change condition,
 $u_3 = v_3 = w_3 = 0$ at $y_3 = hf(x_3, z_3)$, (3.3 a)

$$
u_3 = v_3 = w_3 = 0 \quad \text{at } y_3 = h f(x_3, z_3), \tag{3.3 a}
$$

$$
u_3 = v_3 = w_3 = 0 \text{ at } y_3 = hf(x_3, z_3),
$$

\n
$$
u_3 \to y_3, \quad v_3, \quad w_3, \quad p_3 \to 0 \text{ as } x_3 \to -\infty, \quad z_3 \to \pm \infty,
$$

\n
$$
u_3 \to y_3, \quad w_3 \to 0 \text{ as } y_3 \to \infty,
$$

\n(3.3 c)

$$
u_3 \to y_3, \quad w_3 \to 0 \quad \text{as } y_3 \to \infty,
$$
\n
$$
(3.3c)
$$

 $u_3 \to y_3$, $w_3 \to 0$ as $y_3 \to \infty$,
where $h = A^{1/3} \mu_w^{-1/3} \rho_w^{1/3} ab^{-1/3} \varepsilon^{-1}$ (the subscripts are suppressed below).
The boundary-value problem $(3\ 2)$ – $(3\ 3)$ contains two similarity parameters

here $h = A^{1/3} \mu_{\rm w}^{-1/3} \rho_{\rm w}^{1/3} ab^{-1/3} \varepsilon^{-1}$ (the subscripts are suppressed below).
The boundary-value problem (3.2)–(3.3) contains two similarity parameters, D d h. When the first parameter tends to zero, problem where $h = A^{1/3} \mu_{\rm w}^{-1/3} \rho_{\rm w}^{1/3} a b^{-1/3} \varepsilon^{-1}$ (the subscripts are suppressed below).
The boundary-value problem (3.2)–(3.3) contains two similarity parameters, *D* and *h*. When the first parameter tends to zero, The boundary-value problem (3.2) – (3.3) contains two s
and h. When the first parameter tends to zero, problem (3.2)
boundary-value problem describing two-dimensional flow.
The second similarity parameter is proportio and h . When the first parameter tends to zero, problem (3.2) – (3.3) is reduced to the boundary-value problem describing two-dimensional flow.
The second similarity parameter is proportional to the inertia and viscosi

boundary-value problem describing two-dimensional flow.
The second similarity parameter is proportional to the inertia and viscosity forces
ratio in region 3. For large h values, the disturbed flow near the distortion is at leading order; correspondingly, small h values are connected with the influence of the viscosity force and with linearly disturbed flow near the distortion. ratio in region 3. For large h values, the disturbed flow near the distortion at leading order; correspondingly, small h values are connected with the is the viscosity force and with linearly disturbed flow near the d

4. Linear solution

4. Linear solution
Small values of h correspond to a linear solution which may be sought in the form,

Small values of *h* correspond to a linear solution which may be sought in the form,
\n
$$
u = y + hU + \cdots
$$
, $v = hV + \cdots$, $p = hP + \cdots$, $w = hW + \cdots$.
\n(4.1)
\nCorresponding linearized equations for $D = 1$ have the form,

$$
y\frac{\partial U}{\partial x} + V + \frac{\partial P}{\partial x} = \frac{\partial^2 U}{\partial y^2},\tag{4.2 a}
$$

$$
y\frac{\partial W}{\partial x} + \frac{\partial P}{\partial z} = \frac{\partial^2 W}{\partial y^2},\tag{4.2 b}
$$

$$
y\frac{\partial u}{\partial x} + \frac{\partial v}{\partial z} = \frac{\partial y^2}{\partial y^2},
$$
\n
$$
\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0,
$$
\n(4.2 *c*)

$$
\frac{\partial z}{\partial y} = 0,
$$
\n(4.2 *c*)\n
$$
\frac{\partial P}{\partial y} = 0.
$$
\n(4.2 *d*)

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Equations (4.2) may be transformed to the following equations:

$$
y\frac{\partial S}{\partial x} = \frac{\partial^2 S}{\partial y^2},\tag{4.3 a}
$$

$$
y\frac{\partial S}{\partial x} = \frac{\partial S}{\partial y^2},
$$
\n
$$
-\frac{\partial S(x, 0, z)}{\partial y} = \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial z^2},
$$
\n(4.3*b*)

$$
\int_0^\infty S \, dy = -\frac{\partial f}{\partial x},\tag{4.3 } c
$$

where $S = \partial^2 V / \partial$ $\int_0^{\infty} S dy = -\frac{1}{\partial x}$, (4.3 c)

²V/ ∂y^2 . It may be deduced from the analysis of (4.3) that a non-trivial

he function S exists if $f \neq 0$ or if there is a non-zero convective derivative where $S = \frac{\partial^2 V}{\partial y^2}$. It may be deduced from the analysis of (4.3) that a non-trivial solution for the function S exists if $f \neq 0$ or if there is a non-zero convective derivative of the function S (the wake downstre where $S = \partial^2 V / \partial y^2$. It may be deduced from the analysis of (4.3) that a non-trivial solution for the function S exists if $f \neq 0$ or if there is a non-zero convective derivative of the function S (the wake downstream solution for the function S exists if $f \neq 0$ or if there is a non-zero convective derivative of the function S (the wake downstream from the distortion). Then for an arbitrary surface point, excluding the distortion of the function S (the wake downstream from the distortion). Then for an arbitrary $S(x, y, z) = 0,$ $V(x, y, z) = 0.$ (4.4)

$$
S(x, y, z) = 0, \qquad V(x, y, z) = 0.
$$
\n(4.4)

For the velocity components the following problem may be deduced,

$$
y\frac{\partial U}{\partial x} + \frac{\partial P}{\partial x} = \frac{\partial^2 U}{\partial y^2},\tag{4.5 a}
$$

$$
y\frac{\partial W}{\partial x} + \frac{\partial P}{\partial z} = \frac{\partial^2 W}{\partial y^2},\tag{4.5 b}
$$

$$
\frac{\partial x}{\partial x} + \frac{\partial y}{\partial z} = \frac{\partial y^2}{\partial y^2},
$$
\n
$$
\frac{\partial U}{\partial x} + \frac{\partial W}{\partial z} = 0,
$$
\n(4.5 *c*)

$$
\frac{\partial z}{\partial x} = 0,
$$
\n
$$
\frac{\partial P}{\partial y} = 0,
$$
\n(4.5 *d*)

which describes the quasi-two-dimensional flow outside the distortion and its wake. which describes the quasi-two-dimensional flow outside the distortion and its wake.
Equations (4.5) also follow from the analysis made by Smith (1976) and by Bogolepov
& Lipatov (1985). which describes the
Equations (4.5) also
& Lipatov (1985). $\&$ Lipatov (1985).
5. Analysis of the nonlinearly disturbed flow

Wang (1971) described how to analyse boundary-layer equations. Following Wang, Wang (1971) described how to analyse boundary-layer equations. Following Wang, we will introduce a subcharacteristic surface $\Omega(x, y, z)$ and will transform the inde-Wang (1971) descril
we will introduce a
pendent variables, $x, y, z \rightarrow \Omega(x, y, z), y, z.$ (5.1)

$$
x, y, z \to \Omega(x, y, z), y, z. \tag{5.1}
$$

 $x, y, z \to \Omega(x, y, z), y, z.$ (5.1)
As a result of this analysis we may obtain the following equation, which determines
the subcharacteristics As a result of this analy
the subcharacteristics,

$$
\left(\frac{\partial \Omega}{\partial y}\right)^3 \left[\left(\frac{\partial \Omega}{\partial x}\right)^2 + \left(\frac{\partial \Omega}{\partial z}\right)^2 \right] \left(u\frac{\partial \Omega}{\partial x} + v\frac{\partial \Omega}{\partial y} + w\frac{\partial \Omega}{\partial z}\right) = 0.
$$
 (5.2)

 $\left(\frac{\partial y}{\partial x}\right) \left[\frac{\partial z}{\partial x}\right] + \left(\frac{\partial z}{\partial y}\right) \left[\frac{u \frac{\partial z}{\partial x} + v \frac{\partial y}{\partial y} + w \frac{\partial z}{\partial z}}{v \frac{\partial z}{\partial y}}\right] = 0.$ (5.2)
The first multiplier corresponds to the characteristics of the original equation. The
second multiplier corres The first multiplier corresponds to the characteristics of the original equation. The second multiplier corresponds to the elliptical type of equation for the pressure, and the third multiplier states that the vorticity (The first multiplier corresponds to the characteristics of the original equation. The second multiplier corresponds to the elliptical type of equation for the pressure, and the third multiplier states that the vorticity (v second multiplier corresponds to the elliptical type of equation for the pressure, and the third multiplier states that the vorticity (velocity) field is controlled by streamlines. This form determines the conditions neede *Phil. Trans. R. Soc. Lond.* A (2000)

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boundary-value problem for the three-dimensional boundary layer with compensa-
tion interaction regime. boundary-value problem
tion interaction regime.
Thus there are three in undary-value problem for the three-dimensional boundary layer with compensa-
in interaction regime.
Thus there are three main mechanisms of disturbance propagation in the flow
rresponding to that near a distortion. The fir

tion interaction regime.
Thus there are three main mechanisms of disturbance propagation in the flow
corresponding to that near a distortion. The first mechanism is diffusion, in which
disturbances propagate in a normal di Thus there are three main mechanisms of disturbance propagation in the flow
corresponding to that near a distortion. The first mechanism is diffusion, in which
disturbances propagate in a normal direction with infinite spe corresponding to that near a distortion. The first mechanism is diffusion, in which
disturbances propagate in a normal direction with infinite speed. The second mech-
anism is determined by pressure, in which disturbances disturbances propagate in a normal direction with infinite speed. The second mechanism is determined by pressure, in which disturbances also propagate with infinite speed. The third mechanism is controlled by convection. A only connected with physical processes but are also connected with the mathematispeed. The third mechanism is controlled by convection. All these mechanisms are not
only connected with physical processes but are also connected with the mathemati-
cal formulation of the problem. The mathematical proble only connected with physical processes but are also connected with the mathematical formulation of the problem. The mathematical problem will be well-posed if all mechanisms of disturbance propagation are precisely taken i cal formulation of the problem. The mathematical problem will be well-posed if all
mechanisms of disturbance propagation are precisely taken into account, for example,
in the numerical method. Otherwise, some form of insta mechanisms of disturbance propagation are precisely taken into account, for example,
in the numerical method. Otherwise, some form of instability may be encountered in
the numerical procedure. Analysing the system of equat the numerical procedure. Analysing the system of equations, we may conclude that, as in the linear case, there is a region of zero vertical velocity and a region where this velocity is non zero. the numerical procedure. *i*
as in the linear case, then
this velocity is non zero.
Let us suppose that as i in the linear case, there is a region of zero vertical velocity and a region where
is velocity is non zero.
Let us suppose that, as in the linear case, there is a region of flow with zero vertical
locity near a finite dist

Let us suppose that, as in the linear case, there is a region of flow with zero vertical velocity near a finite distortion. Then the disturbed flow is described by the following equations:

$$
u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} + \frac{\partial p}{\partial x} = \frac{\partial^2 u}{\partial y^2},
$$
\n(5.3*a*)

$$
u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} + \frac{\partial p}{\partial z} = \frac{\partial^2 w}{\partial y^2},\tag{5.3 b}
$$

$$
\frac{\partial}{\partial z} + \frac{\partial}{\partial z} = \frac{\partial y^2}{\partial y^2},
$$
\n
$$
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0.
$$
\n(5.3 c)

Equations (5.3) may be transformed as follows,

$$
u\frac{\partial \omega_y}{\partial x} + w\frac{\partial \omega_y}{\partial z} = \frac{\partial^2 \omega_y}{\partial y^2},\tag{5.4}
$$

where

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$$
\omega_y = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}.
$$

 $\omega_y = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}$.
Equation (5.4) is accompanied by uniform boundary conditions,

by uniform boundary conditions,
\n
$$
\omega_y = 0 \quad \text{at } y = 0,
$$
\n(5.5)

$$
\omega_y = 0 \quad \text{at } y = 0,
$$

\n
$$
\omega_y \to 0 \quad \text{as } y \to \infty.
$$

\n(5.5)
\n(5.6)

Then a zero solution for vorticity component $\omega_y \to 0$ as $y \to \infty$. (5.6)
if if bonent ω_y exists for the region of now analysed
 $\omega_y = 0.$ (5.7)

$$
\omega_y = 0. \tag{5.7}
$$

 $\omega_y = 0.$ (5.7)
Applying the divergence operator to the equations (5.3) allows us to obtain the following equation: Applying the diverg
following equation:

$$
\frac{\partial u}{\partial x}\frac{\partial u}{\partial x} + 2\frac{\partial w}{\partial x}\frac{\partial u}{\partial z} + \frac{\partial w}{\partial z}\frac{\partial w}{\partial z} = -\frac{\partial^2 p}{\partial x^2} - \frac{\partial^2 p}{\partial z^2}.
$$
(5.8)

Equations (5.3) and (5.4) allow to transform (5.7) as follows:

$$
\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2 = -\frac{1}{2}\Delta p.
$$
\n(5.9)

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Only the zero solution for the velocity component then exists. But this conclusion Only the zero solution for the velocity component then exists. But this conclusion contradicts equation $(5.3 b)$. Therefore, in the nonlinear case, the suggestion that the normal velocity is zero in part of the disturbed Only the zero solution for the velocity component then exists. But this conclusion contradicts equation $(5.3 b)$. Therefore, in the nonlinear case, the suggestion that the normal velocity is zero in part of the disturbed contradicts equation $(5.3 b)$. There
normal velocity is zero in part of
fulfilled only for the linear case. fulfilled only for the linear case.
6. Numerical solution

5. Numerical solution
The spectral method originally suggested by Duck & Burggraf (1986) was used to
solve the problem $(3\ 2)-(3\ 3)$ Prandtl's transposition The spectral method originally suggested by Duck $\&$ Esolve the problem $(3.2)-(3.3)$. Prandtl's transposition, solve the problem (3.2) – (3.3) . Prandtl's transposition,

$$
\tilde{x} = x, \quad \tilde{y} = y - f(x, z), \quad \tilde{z} = z, \quad \tilde{u} = u - \tilde{y}, \tag{6.1 a}
$$

$$
\tilde{v} = v - u \frac{\partial f}{\partial x} - w \frac{\partial f}{\partial z}, \quad \tilde{w} = w, \quad \tilde{p} = p,
$$
\n(6.1*b*)

leads to the problem including unchanged equations and the following boundary conditions: $u = v = w = 0$ at $y = 0$; (6.2 a)

$$
u = v = w = 0 \text{ at } y = 0,
$$
\n(6.2*a*)

$$
u = v = w = 0 \quad \text{at } y = 0,
$$

(6.2*a*)

$$
u \to 0, \quad v, w, p \to 0 \quad \text{as } x \to -\infty \text{ and as } z \to \pm \infty,
$$

(6.2*b*)

$$
u = v = w = 0 \quad \text{at } y = 0,
$$

\n
$$
v, w, p \to 0 \quad \text{as } x \to -\infty \text{ and as } z \to \pm \infty,
$$

\n
$$
u \to f(x, z), \quad w \to 0 \quad \text{as } y \to \infty.
$$

\n(6.2 c)

 $u \to f(x, z)$, $w \to 0$ as $y \to \infty$. (6.2 c)
Fourier transforms in the x- and z-directions give the following expression for the
longitudinal velocity. Fourier transforms in t
longitudinal velocity:

longitudinal velocity:
\n
$$
u^{**}(k,l,y) = \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} u(x,y,z) \exp(-ikx - ilz) dx dz.
$$
\n(6.3)
\nThe system to be solved is written then as follows:

$$
it then the mass follows:
$$

\n
$$
iku^{**} + v^{**'} + ilw^{**} = 0,
$$
\n(6.4 a)

$$
iku^{**} + v^{**'} + ilw^{**} = 0,
$$
\n(6.4 a)
\n
$$
u^{**\prime} - ikyu^{**} - v^{**} - ikP = \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)^{**} = R_1^{**},
$$
\n(6.4 b)

$$
u^{***'} - ikyu^{**} - v^{**} - ikP = \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right) = R_1^{**},\tag{6.4b}
$$

$$
w^{**} - ikyw^{**} - iIP^{**} = \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)^{**} = R_2^{**}.\tag{6.4c}
$$

The boundary conditions include

include
\n
$$
u^{**} = v^{**} = w^{**} = 0
$$
 at $y = 0$, (6.5 a)
\n $u^{**} = u^{**} = u^{**} = 0$

$$
u^{**} = v^{**} = w^{**} = 0 \text{ at } y = 0,
$$

\n
$$
u^{**} \to F^{**}, \quad w^{**} \to 0 \text{ as } y \to \infty.
$$
\n(6.5 a)
\n(6.5 b)

The following procedure is mainly the same as that described by Duck & Burggraf The following procedure is mainly the same as that described by Duck & (1986). The distortion analysed has the geometry $f(x, z) = h \exp(-(x^2 +$
numerical grid includes $NX \times NY \times NZ = 64 \times 26 \times 32$ nodes with $(x^2 + z^2)$). $\frac{1}{2}$). The
extens The following procedure is mainly the same as that described by Duck & Burggraf (1986). The distortion analysed has the geometry $f(x, z) = h \exp(-(x^2 + z^2))$. The numerical grid includes $NX \times NY \times NZ = 64 \times 26 \times 32$ nodes, with the st (1986). The distortion analysed has the geometry $f(x, z) = h \exp(-(x^2 + z^2))$. The numerical grid includes $NX \times NY \times NZ = 64 \times 26 \times 32$ nodes, with the steps for corresponding coordinates $\Delta x = \Delta z = 0.3$, $\Delta y = 0.4$, $D = 1$. To check o numerical grid includes $NX \times NY \times NZ = 64 \times 26 \times 32$ nodes, with the steps
for corresponding coordinates $\Delta x = \Delta z = 0.3$, $\Delta y = 0.4$, $D = 1$. To check out the
accuracy of results, the grid $NX \times NY \times NZ = 128 \times 26 \times 64$ was used alon for corresponding coordinates $\Delta x = \Delta z = 0.3$, $\Delta y = 0.4$, $D = 1$. To check out the accuracy of results, the grid $NX \times NY \times NZ = 128 \times 26 \times 64$ was used along with non-
uniform spacing in the Y-direction. The results obtained o accuracy of results, the grid $NX \times NY \times NZ = 128 \times 26 \times 64$ was used along with non-
uniform spacing in the Y-direction. The results obtained on both grids were almost
the same. To have a uniformly valid solution of linearized uniform spacing in the Y-direction. The results obtained on both grids were almost
the same. To have a uniformly valid solution of linearized equations, integration in
spectral space was fulfilled by analytical presentati the same. To have a uniformly valid solution of linearized equations, integration in

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Figure 1. Pressure distributions in the symmetry plane.

In figure 1 the pressure disturbance distributions are presented in the symmetry plane for different h values. The negative h values correspond to the hollow on the In figure 1 the pressure disturbance distributions are presented in the symmetry plane for different h values. The negative h values correspond to the hollow on the surface. It may be seen that the distortion depth in plane for different h values. The negative h values correspond to the hollow on the surface. It may be seen that the distortion depth increase leads to the maximum pressure disturbance rise. The distributions presente pressure disturbance rise. The distributions presented are characterized by two local minima located downstream and upstream from the coordinate origin.

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Figure 4. Pressure distributions in the transversal direction.

Figure 4. Pressure distributions in the transversal direction.
It is worth mentioning that there is tendency in the pressure distribution to form
plateau region It is worth mentioning that there is tendency in the pressure distribution to form
plateau region.
Results of the longitudinal skin friction calculations are presented on figure 2. It
ay be concluded that the distortion de

a plateau region.
Results of the longitudinal skin friction calculations are presented on figure 2. It a plateau region.
Results of the longitudinal skin friction calculations are presented on figure 2. It
may be concluded that the distortion depth increase leads to the minimum skin
friction diminishing. A limiting depth va Results of the longitudinal skin friction calculations are presented on figure 2. It
may be concluded that the distortion depth increase leads to the minimum skin
friction diminishing. A limiting depth value exists for whi friction diminishing. A limiting depth value exists for which the longitudinal skin friction equals zero at one surface point.

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Figure 5. Vertical velocity distributions.

Figure 5. Vertical velocity distributions.
The results presented in figure 3 illustrate the minimum longitudinal skin friction
pendence on the parameter h . It may be supposed that, for large depth values, a The results presented in figure 3 illustrate the minimum longitudinal skin friction dependence on the parameter h. It may be supposed that, for large depth values, a limiting regime exists for which minimal longitudinal s The results presented in figure 3 illustrate the minimum longitudinal skin friction dependence on the parameter h . It may be supposed that, for large depth values, a limiting regime exists for which minimal longitudinal dependence on the parameter h . It may be supposed that, for large depth values, a limiting regime exists for which minimal longitudinal skin friction tends to a finite value. The distortion height increase leads to a di limiting regime exists for which minimal longitudinal skin friction tends to a finite value. The distortion height increase leads to a different tendency. At the same time,

The disturbed flow near finite distortions of the form

sturbed flow near finite distortions of the form
\n
$$
f(x, z) = 0
$$
, if $R \ge 1$ and $f(x, z) = h \cos^2(\pi R/2)$ for $R < 1$,
\n $= x^2 + z^2$, was also investigated.
\nare 4 the dependence of pressure disturbance is presented as a functor

where $R = x^2 + z^2$, w

In figure 4 the dependence of pressure disturbance is presented as a function of where $R = x^2 + z^2$, was also investigated.
In figure 4 the dependence of pressure disturbance is presented as a function of
z for $h = 1.5$ in the linear and nonlinear case (the solid line corresponds to the
nonlinear case In figure 4 the dependence of pressure disturbance is presented as a function of z for $h = 1.5$ in the linear and nonlinear case (the solid line corresponds to the nonlinear case) for different longitudinal coordinate v z for $h = 1.5$ in the linear and nonlinear case (the solid line corresponds to the nonlinear case) for different longitudinal coordinate values. It may be seen that, in accordance with the aforementioned results, the dist nonlinear case) for different longitudinal coordinate values. It may be seen that, in accordance with the aforementioned results, the distortion influence is revealed in the full flow field, due to the ellipticity of the e

In the state of results, the distortion influence is revealed in the full flow field, due to the ellipticity of the equation for the pressure disturbance.
In figure 5 the vertical velocity distribution is presented in the the full flow field, due to the ellipticity of the equation for the pressure disturbance.
In figure 5 the vertical velocity distribution is presented in the plane parallel to the
flat plate. It may be seen in fact that ou In figure 5 the vertical velocity distribution is presented in the plane parallel to the flat plate. It may be seen in fact that outside the influence zone the vertical velocity almost equals zero. This result does not co flat plate. It may be seen in fact that outside the influence zone the vertical velocity almost equals zero. This result does not contradict to the conclusion made in $\S 5$. When the distortion height is not so large, non almost equa
When the c
are small.
It is wort hen the distortion height is not so large, nonlinear effects outside the distortion
as small.
It is worth mentioning that the mathematical model investigated may be used in
wide class of problems. The results obtained for

are small.
It is worth mentioning that the mathematical model investigated may be used in
a wide class of problems. The results obtained, for example, may be used to estimate It is worth mentioning that the mathematical model investigated may be use
a wide class of problems. The results obtained, for example, may be used to estin
the effectiveness of flow control in three-dimensional laminar bo

the effectiveness of flow control in three-dimensional laminar boundary layers.
The work was done under the financial help of the Russian Foundation of Basic Research (grant
number 96.01.01537) The work was done under the financial help of the Russian Foundation of Basic Research (grant number 96-01-01537).

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